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27. (II) (*a*) In Fig. 30–28a, assume that the switch S has been in position A for sufficient time so that a steady current $I_0 = V_0/R$ flows through the resistor R. At time $t = 0$, the switch is quickly switched to position B and the current through R decays according to $I = I_0 e^{-t/\tau}$. Show that the maximum emf $\mathscr{E}_{max}$ induced in the inductor during this time period equals the battery voltage V_0. (*b*) In Fig. 30–28b, assume that the switch has been in position A for sufficient time so that a steady current $I_0 = V_0/R$ flows through the resistor R. At time $t = 0$, the switch is quickly switched to position B and the current decays through resistor R' (which is much greater than R) according to $I = I_0 e^{-t/\tau'}$. Show that the maximum emf $\mathscr{E}_{max}$ induced in the inductor during this time period is $(R'/R)V_0$. If $R' = 55R$ and $V_0 = 120\,\text{V}$, determine $\mathscr{E}_{max}$. [When a mechanical switch is opened, a high-resistance air gap is created, which is modeled as R' here. This Problem illustrates why high-voltage sparking can occur if a current-carrying inductor is suddenly cut off from its power source.]

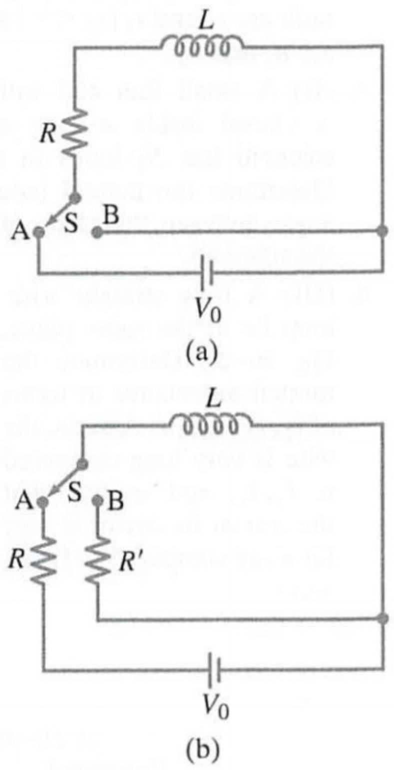

FIGURE 30–28
Problem 27.

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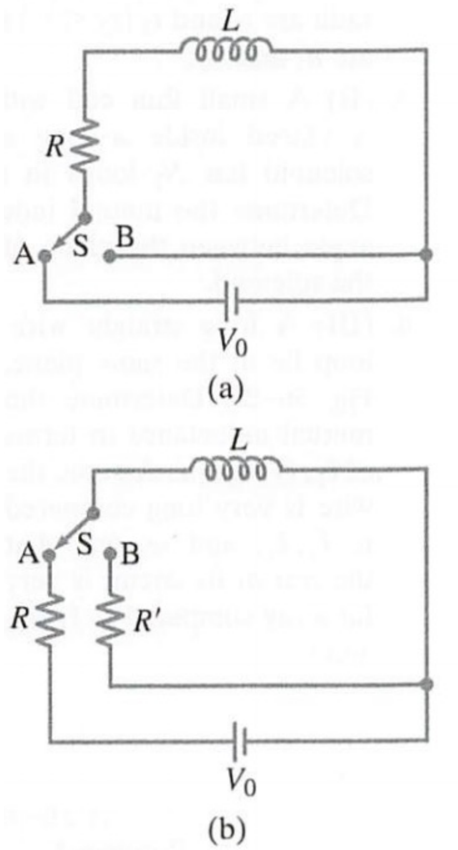

a) $-L\frac{dI}{dt}$

$I = I_0 e^{-t/\tau}$
$= I_0 e^{-tR/L}$

$\mathcal{E} = -L\frac{dI}{dt} = -L\frac{d}{dt}(I_0 e^{-t/\tau})$

$= \frac{+LI_0R}{L} e^{-t/\tau} = \underbrace{I_0R}_{V_0} e^{-t/\tau}$

$\mathcal{E}_{max}$ when $t=0$

$\mathcal{E}_{max} = V_0$

b) $V_0 = I_0R \Rightarrow I_0 = \frac{V_0}{R}$

$\mathcal{E} - IR' = 0$ $\quad\quad \mathcal{E} = IR' = R'I_0 e^{-t/\tau'} = \frac{R'V_0}{R} e^{-t/\tau'}$

$\mathcal{E}_{max} = \frac{R'V_0}{R} = \frac{55R}{R}(120V) = 6.6\,kV$