Ch. 30 – 75

75. A pair of straight parallel thin wires, such as a lamp cord, each of radius r, are a distance ℓ apart and carry current to a circuit some distance away. Ignoring the field within each wire, show that the inductance per unit length is $(\mu_0/\pi)\ln[(\ell - r)/r]$.

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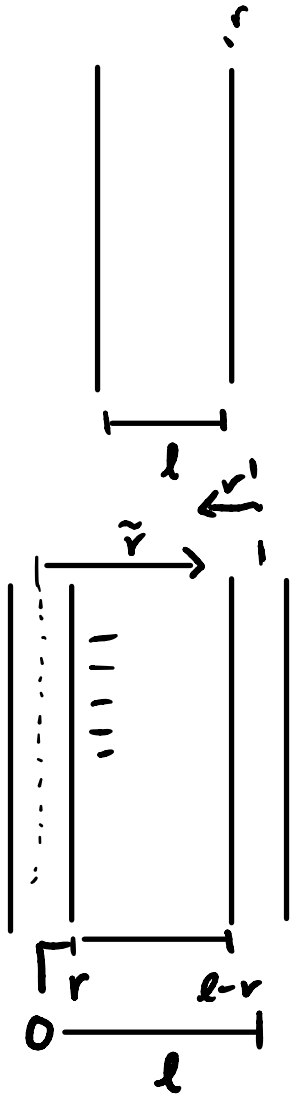
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$



$$L = \frac{\Phi_B}{I} = \frac{1}{I}\int_r^{l-r}\left[\frac{\mu_0 I}{2\pi\tilde{r}} + \frac{\mu_0 I}{2\pi(l-\tilde{r})}\right] h\, d\tilde{r}$$

$$= \frac{\mu_0 h}{2\pi}\int_r^{l-r}\frac{1}{\tilde{r}} + \frac{1}{(l-\tilde{r})}\, d\tilde{r} = \frac{\mu_0 h}{2\pi}\left[\ln(\tilde{r}) - \ln(l-\tilde{r})\right]\Big|_r^{l-r}$$

$$\frac{L}{h} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{l-r}{r}\right) - \ln\left(\frac{r}{l-r}\right)\right]$$

$$\frac{L}{h} = \frac{\mu_0}{2\pi}\ln\left[\left(\frac{l-r}{r}\right)^2\right] = \frac{\mu_0}{\pi}\ln\left(\frac{l-r}{r}\right)$$

$$\frac{\mu_0 I}{2\pi\tilde{r}} + \frac{\mu_0 I}{2\pi r'}$$

$$\tilde{r} + r' = l$$