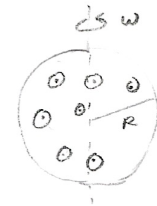
January 2013

Ring radius r
conductivity σ
density φ

Immersed in field $\vec{B}$

Ring rotating → Find ω assuming ω_0 at $t=0$



magnetic flux $\Phi_B = \vec{A}\cdot\vec{B} = \pi r^2 \cdot B\cos\theta$ for $\theta = \omega t$

$$\frac{-d\Phi_B}{dt} = \mathcal{E} = \pi r^2 B \sin(\omega t)\cdot\omega$$

$P = IV = I\pi r^2 B\sin(\omega t)\omega$

to find I: $I = J\cdot A$

$J = \sigma\vec{E} \rightarrow \sigma\frac{\mathcal{E}}{2\pi r}$

$\mathcal{E} = \oint \vec{E}\cdot d\ell = 2\pi r E$

$E = \frac{\mathcal{E}}{2\pi r}$

$$= \frac{\sigma A}{2\pi r}\left[\pi r^2 \omega B \sin(\omega t)\right]^2$$

$$= \frac{A\sigma\pi^2 r^4 \omega^2 B^2 \sin^2(\omega t)}{2\pi r} = \frac{A\sigma\pi r^3\omega^2 B^2 \sin^2(\omega t)}{2}$$

$$P = \frac{-dE}{dt} = \frac{-d}{dt}\left(\frac{1}{2}I\omega^2\right) = -\frac{d}{dt}\left(\frac{1}{2}Mr^2\omega^2\right) = -Mr^2\omega\frac{d\omega}{dt}$$

$I = \frac{1}{2}Mr^2$

$\varphi = \frac{M}{V}$

$M = \varphi\cdot A\cdot 2\pi r$

$$-r^2\omega\frac{d\omega}{dt}\varphi A\cdot 2\pi r = \frac{1}{2}A\sigma\pi r^3\omega^2 B^2\sin^2(\omega t)$$

$$-\frac{1}{\omega}d\omega = \frac{1}{4}\frac{rB^2}{\varphi}\sin^2(\omega t)\,dt = \frac{rB^2}{4\varphi}\sin^2(\omega t)\,dt$$

$$\langle P\rangle = \frac{A\sigma}{2}\pi r^3\omega^2 B^2\cdot\frac{1}{2} = \frac{A\sigma}{4}\pi r^3\omega^2 B^2 = -\varphi A(2\pi)r^3\omega\frac{d\omega}{dt}$$

$$dt\,\frac{-\sigma}{4}B^2 = 2\varphi\frac{1}{\omega}\,d\omega$$

$$dt\,\frac{-\sigma B^2}{8\varphi} = \frac{1}{\omega}\,d\omega$$

$$\omega(t) = \omega_0 e^{-\sigma B^2/8\varphi}$$