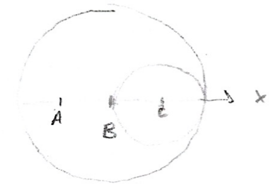
EM-2 January 2016

Uniformly charged sphere radius R

volume charge density φ

Carve out spherical radius $R/2$

Calculate $\vec{E}$ at A, B, C



Volume of sphere: $\frac{4}{3}\pi r^3$

Q big sphere $\Rightarrow$ $\frac{4}{3}\pi R^3$ Q small sphere. $-\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = -\frac{1}{6}\pi R^3$

Total $Q = \left[\frac{4}{3} - \frac{1}{6}\right](\pi R^3) = \frac{7}{6}\pi R^3$

Big sphere:

$$\int E \cdot dA = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E_B \cdot 4\pi r^2 = \frac{(4\pi r^3)\varphi}{3\varepsilon_0} \Rightarrow E_B = \frac{r\varphi}{3\varepsilon_0}$$

Small sphere

$$\int E \cdot dA = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E_S \cdot 4\pi r^2 = \frac{-(4\pi r^3)\varphi}{3\varepsilon_0} \Rightarrow E_S = \frac{-r\varphi}{3\varepsilon_0} \quad \text{for} \quad r < \frac{R}{2}$$

$$E_S \cdot 4\pi r^2 = \frac{-\pi R^3\varphi}{6\varepsilon_0} \Rightarrow E_S = \frac{-R^3\varphi}{24 r^2\varepsilon_0}$$

$\rightarrow$ At A: $r_B = \frac{R}{2}$ $r_S = R$

$$E_B = \left(\frac{R}{2}\right)\frac{\varphi}{3\varepsilon_0} = \frac{R\varphi}{6\varepsilon_0} \qquad E_S = \frac{-R^3\varphi}{24\varepsilon_0 R^2} = \frac{-R\varphi}{24\varepsilon_0}$$

$$E_B(-\hat{r}) + E_S(-\hat{r}) = -(E_B + E_S)\hat{r}$$

$$\vec{E}_A = -\left(\frac{1}{6} - \frac{1}{24}\right)\frac{R\varphi}{\varepsilon_0} = -\left(\frac{4-1}{24}\right)\frac{R\varphi}{\varepsilon_0} = \frac{-R\varphi}{8\varepsilon_0}\hat{r}$$

$\rightarrow$ At B: $r_B = 0$ $r_S = \frac{R}{2}$

$$E_B = 0 \qquad E_S = \frac{-R\varphi}{6\varepsilon_0}$$

$$E_S(-\hat{r}) = -E_S\hat{r}$$

$$\vec{E}_B = \frac{R\varphi}{6\varepsilon_0}\hat{r}$$

$\rightarrow$ At C: $r_B = \frac{R}{2}$ $r_S = 0$

$$E_B = \frac{R\varphi}{6\varepsilon_0} \qquad E_S = 0 \qquad E_B\hat{r}$$

$$\vec{E}_B = \frac{R\varphi}{6\varepsilon_0}\hat{r}$$