CM-2 January 2019

2 beads mass m on circular ring

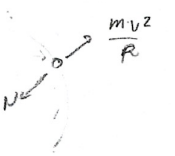
ring mass M & radius R

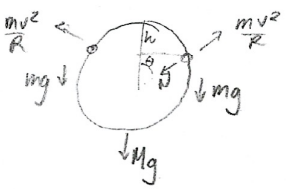
a. Find first time $T=0$

Gravitational force at $t=0$

$\downarrow Mg + 2mg$

Angular force:





$N = \frac{mv^2}{R} - mg\cos\theta$

(x-components cancel out by geometry)

$mgh = \frac{1}{2}mv^2 \Rightarrow mv^2 = 2mgh$

$mv^2 = 2mg(1-\cos\theta)R$

$h = (1-\cos\theta)R$

$N = -2mg(1-\cos\theta) + mg\cos\theta$

$= -2mg + 3mg\cos\theta$

$\rightarrow$ y component: $N\cos\theta$

$(2mg + 3mg\cos\theta)\cos\theta$

$mg(-2+3\cos\theta)\cos\theta$

$T = Mg + 2mg\cos\theta(-2+3\cos\theta)$

$= 6mg\cos^2\theta - 4mg\cos\theta + Mg$

$= \cos^2\theta - \frac{2}{3}\cos\theta + \frac{M}{6m}$

$\rightarrow \cos\theta = \frac{+\frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{M}{6m}}}{2}$

$= \frac{1}{3} \pm \frac{1}{3}\sqrt{1 - \frac{9M}{6m}} = \frac{1}{3}\left(1 \pm \sqrt{1 - \frac{3}{2}\frac{M}{m}}\right)$

$\theta = \arccos\left(\frac{1}{3}\left[1 \pm \sqrt{1 - \frac{3}{2}\frac{M}{m}}\right]\right)$

b. What condition must $\frac{M}{m}$ satisfy for $T \to 0$?

→ square root has to be real.

$$1 - \frac{3}{2}\frac{M}{m} \geq 0$$

$$-\frac{M}{m} \geq \frac{2}{3}$$

$$\frac{M}{m} \leq \frac{2}{3}$$