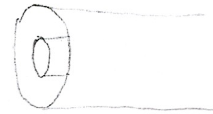
# January 2019

Coaxial cable length $l$

inner radius $a$ → charge density $\lambda$

outer shell radius $b$ → charge density $-\lambda$



a. Determine $\vec{E}$ everywhere.

$$\int \vec{E} \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$


region I: $r < a$

$\vec{E} = 0$

no fields inside a conductor.

region II: $a < r < b$

$q_{enclosed} = \lambda \cdot l$

$$E \cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

region III: $r > b$

$q_{enclosed} = (\lambda \cdot l) + (-\lambda \cdot l) = 0$

$\vec{E} = 0$.

b. Calculate electrostatic energy stored.

$$U = \frac{\varepsilon_0}{2} \int E^2 \, dV = \int_0^L \int_0^{2\pi} \int_a^b \frac{\lambda^2}{4\pi^2 r^2 \varepsilon_0^2} (r\,d\theta)\, dr\, dz = \frac{\lambda^2 \varepsilon_0}{8\pi^2 \varepsilon_0^2} \int_a^b \frac{1}{r}\, dr \; 2\pi \int_0^l dz = \frac{\lambda^2 l \varepsilon_0}{4\pi \varepsilon_0^2} \ln\left(\frac{a}{b}\right)$$

$$= \frac{\lambda^2 l}{4\pi\varepsilon_0} \ln\left(\frac{a}{b}\right)$$

c. Now they carry currents in opposite directions. Find $\vec{B}$ everywhere.

$$\int B \cdot dl = \mu_0 I$$

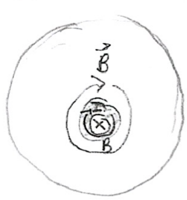

region I: $r < a$

$$I_{enclosed} = \frac{\pi I r^2}{\pi a^2}$$

$$B \cdot 2\pi r = \frac{\mu_0 I r^2}{a^2}$$

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

$$\vec{B} = \frac{\mu_0 I r}{2\pi a^2} \hat{\phi}$$




Region II: $a < r < b$

$I_{enclosed} = I$

$$B \cdot 2\pi r = I \mu_0$$

$$B = \frac{I\mu_0}{2\pi r}$$

$$\vec{B} = \frac{I\mu_0}{2\pi r} \hat{\phi}$$

region III: $r > b$

$I_{enclosed} = I + -I = 0$

$\vec{B} = 0$.

d. Total magnetic energy store.

$$U = \frac{1}{2\mu_0}\int |B|^2 dV = \int_0^{l}\int_0^{2\pi}\int_0^{a} \frac{\mu_0^2 I^2 r^2}{4\pi^2 a^4} \cdot r \, dr\, d\theta\, dz + \int_0^{l}\int_0^{2\pi}\int_a^{b} \frac{\mu_0^2 I^2}{4\pi r^2} r\, dr\, d\theta\, dz = \frac{\pi l}{\mu_0}\left[\frac{\mu_0^2 I^2}{4\pi^2 a^4}\int_0^a r^3\, dr + \frac{\mu_0^2 I^2}{4\pi^2}\int_a^b \frac{1}{r}\, dr\right]$$

$$\frac{r^4}{4} \qquad \ln(b/a)$$

$$= \frac{\pi l}{\mu_0}\left[\frac{\mu_0^2 a^4 I^2}{16\pi^2 a^4} + \frac{\mu_0^2 I^2}{4\pi^2}\ln\left(\frac{b}{a}\right)\right] = \frac{\mu_0 I^2 l}{4\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right]$$

Self inductance.

$U = \frac{1}{2} L I^2$ where $L$ is self-inductance.

$$\frac{2U}{I^2} = L$$

$$\frac{2U}{I^2} = \frac{\mu_0 l}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right]$$

Timed ~ 25 minutes