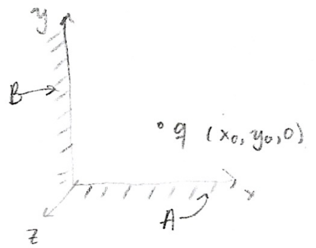
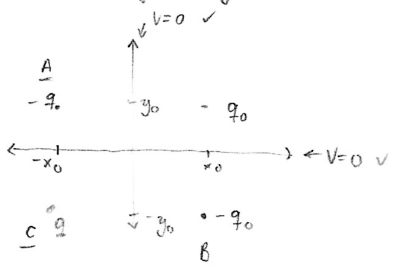
September 2013



$\rightarrow$ A & B are grounded conducting plates

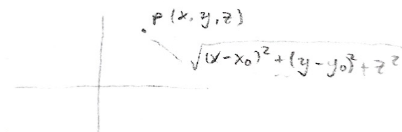
a). $x_0 = \sqrt{3}\, d$ and $y_0 = d$. Find $\Phi(x,y,z)$

$\rightarrow$ method of images $\rightarrow$ at interface (surface of conducting plates) $V=0$



By adding image charges A & B we get zero at x-axis and y-axis but they interact which breakes one condition as it fulfills the other. By adding charge C we can counter this and get the desired boundary conditions at interfaces

Φ due to point charge: $\Phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$



$$\Phi = \Phi_A + \Phi_B + \Phi_C + \Phi_q = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+z^2}} - \frac{1}{\sqrt{(x+x_0)^2+(y-y_0)^2+z^2}} + \dots\right.$$

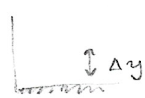
$$\left.\dots + \frac{1}{\sqrt{(x+x_0)^2+(y+y_0)^2+z^2}} - \frac{1}{\sqrt{(x-x_0^2)+(y+y_0)^2+z^2}}\right]$$

$x_0 = \sqrt{3}$ & $y_0 = 1$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{\sqrt{(x-\sqrt{3})^2+(y-1)^2+z^2}} - \frac{1}{\sqrt{(x+\sqrt{3})^2+(y-1)^2+z^2}} + \frac{1}{\sqrt{(x+\sqrt{3})^2+(y+1)^2+z^2}} - \frac{1}{\sqrt{(x-\sqrt{3})^2+(y+1)^2+z^2}}\right\}$$

b. Find surface charge density of A

$\sigma = \varepsilon_0 E$ $\qquad E_y = -\frac{\partial\Phi}{\partial y}$



$\rightarrow$ only care about y and evaluate at $y=0$

$$= -\varepsilon_0 \left.\frac{\partial\Phi}{\partial y}\right|_{y=0}$$

$$= -\varepsilon_0\frac{\partial}{\partial y}\left\{\frac{q}{4\pi\varepsilon_0}\left[(x-\sqrt{3})^2+(y-1)^2+z^2\right]^{-1/2} - \left[(x+\sqrt{3})^2+(y-1)^2+z^2\right]^{-1/2} + \left[(x+\sqrt{3})^2+(y+1)^2+z^2\right]^{-1/2} - \left[(x-\sqrt{3})^2+(y+1)^2+z^2\right.\right.$$

$$= \frac{-q}{4\pi}\left(-\frac{1}{2}\right)\left\{\left[(x-\sqrt{3})^2+(y-1)^2+z^2\right]^{-3/2}\cdot 2(y-1) - \left[(x+\sqrt{3})^2+(y-1)^2+z^2\right]^{-3/2}\cdot 2(y-1) + \left[(x+\sqrt{3})^2+(y+1)^2+z^2\right]^{-3/2}\cdot 2(y+1) - \left[(x-\sqrt{3})^2+(y+1)^2+z^2\right]^{-3/2}\cdot 2(y+\right.$$

evaluate at $y=0$ $\rightarrow$

$$= \frac{q}{4\pi}\left\{\frac{-1}{\left[(x-\sqrt{3})^2+1+z^2\right]^{3/2}} + \frac{1}{\left[(x+\sqrt{3})^2+1+z^2\right]^{3/2}} + \frac{1}{\left[(x+\sqrt{3})^2+1+z^2\right]^{3/2}} + \frac{-1}{\left[(x-\sqrt{3})^2+1+z^2\right]^{3/2}}\right\}$$

$$= \frac{q}{2\pi}\left\{\frac{-1}{\left[(x-\sqrt{3})^2+z^2+1\right]^{3/2}} + \frac{1}{\left[(x+\sqrt{3})^2+z^2+1\right]^{3/2}}\right\}$$

units: $\frac{[Q][m]}{[m]^3} = \frac{C}{[m^2]}$ correct units.

set $d=1$ at top so we have $\frac{[m]}{[m^3]}$ in parentheses and $[Q]$ outside.