CM-2 September 2014

$V = \frac{kr^2}{2}$ for $k > 0$

$T = \frac{1}{2} m v^2$

$\mathcal{L} = T - V = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2} k (x^2 + y^2) = \frac{1}{2} m \dot{r}^2 - \frac{1}{2} k r^2$

$\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$ $\qquad \frac{\partial \mathcal{L}}{\partial x} = -kx$ $\qquad \rightarrow \ddot{r} = -\frac{k}{m} r$

$\ddot{x} = -\frac{k}{m} x$

$\ddot{y} = -\frac{k}{m} y$

Option 1: $x = a\cos\theta = y$

$x = \cos\left(\sqrt{\frac{k}{m}}\, t\right)$ and $y = \cos\left(\sqrt{\frac{k}{m}}\, t\right)$ $\rightarrow$ straight line

$x^2 + y^2 = 2\cos^2\left(\sqrt{\frac{k}{m}}\, t\right)$

Option 2: $x = a\cos\theta$
$y = a\sin\theta$

$x = \cos\left(\sqrt{\frac{k}{m}}\, t\right)$ and $y = \sin\left(\sqrt{\frac{k}{m}}\, t\right)$ $\rightarrow$ ellipse

$x^2 + y^2 = 1$

For a linear combination of sin and cos:
we get an ellipse unless coefficients are the same (where we get a straight line).