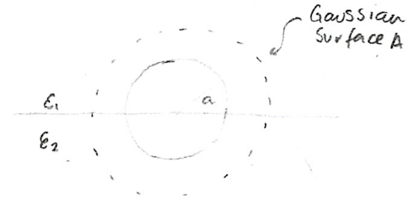
September 2014

Charged metal sphere
radius a
total charge q



a. Find surface charge density σ

Surface area half sphere: $2\pi a^2$

$$\sigma_1 = D_1 = \varepsilon_1 \vec{E}_1 = \frac{Q\varepsilon_1}{2\pi(\varepsilon_1+\varepsilon_2)a^2} = \sigma_1$$

(Note: I did part b first and then plugged for E here

$$\sigma_2 = \frac{Q\varepsilon_2}{2\pi(\varepsilon_1+\varepsilon_2)a^2}$$

Check: $\sigma_1 + \sigma_2 = \frac{Q}{2\pi r^2} \Rightarrow \frac{Q(\varepsilon_1+\varepsilon_2)}{2\pi(\varepsilon_1+\varepsilon_2)r^2} = \frac{Q}{2\pi a^2}$ ✓

b. Find electric field everywhere:

$$\int_S \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon}$$ and $\vec{E}_1 = \vec{E}_2$ at boundary. $\vec{E}_T = \vec{E}_1 = \vec{E}_2$

$$\int_S \vec{D}\cdot da = Q_{enc} = D_1\, 2\pi r^2 + D_2\, 2\pi r^2 = Q$$

$D_1 = E_1\varepsilon_1$ & $D_2 = E_2\varepsilon_2$

$$E_1\varepsilon_1 + E_2\varepsilon_2 = \frac{Q}{2\pi r^2} \Rightarrow E_T(\varepsilon_1+\varepsilon_2) = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)r^2}$$ $\rightarrow$ for $r>a$

for $r<a$ $Q_{enclosed} = 0$

$\vec{E} \rightarrow$ radially outward

$$\vec{E} = \begin{cases} 0\,\hat{r} & \text{for } r<a \\ \dfrac{Q}{2\pi(\varepsilon_1+\varepsilon_2)r^2}\,\hat{r} & \text{for } r>a. \end{cases}$$

c. Find surface potential assuming zero at infity

$$\Phi_E = -\int_r^{\infty} \vec{E} \cdot d\vec{l}$$

$$= -\int_a^{\infty} \frac{Q \, dr}{2\pi r^2(\varepsilon_1+\varepsilon_2)} = \frac{-Q}{2\pi(\varepsilon_1+\varepsilon_2)} \underbrace{\int_a^{\infty} r^{-2} \, dr}_{-r^{-1}}$$

$$= \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)} \left. \frac{-1}{r} \right|_a^{\infty} = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)a}$$

$$\Phi_E = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)a}$$ → surface potential

d. Find bound charge density of dielectric just outside the sphere

$\sigma_B = \vec{P} \cdot \hat{n}$ where P is the polarization

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$ χ_e → dielectric susceptibility

$\vec{P}_1 = \varepsilon_0 \left(\frac{\varepsilon_1}{\varepsilon_0} - 1\right)\vec{E}$ $\varepsilon_r - 1 = \chi_e$ for us $\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_0}$ or $\frac{\varepsilon_2}{\varepsilon_0}$ (relative permithvity)

$= (\varepsilon_1 - \varepsilon_0)\vec{E}$

$\vec{P}_2 = (\varepsilon_2 - \varepsilon_0)\vec{E}$ Evaluate $\vec{E}$ at a (since we're just outside a)

$$\sigma_{B1} = (\varepsilon_1 - \varepsilon_0)E = \frac{-(\varepsilon_1 - \varepsilon_0)Q}{2\pi a^2(\varepsilon_1+\varepsilon_2)}$$ → above x axis

$$\sigma_{B2} = (\varepsilon_2 - \varepsilon_0)E = \frac{-(\varepsilon_2 - \varepsilon_0)Q}{2\pi a^2(\varepsilon_1+\varepsilon_2)}$$ → below x axis