EM-1 September 2016

Disc radius R → in x-y plane.

Surface charge density σ center at $(x_0, 0, 0)$

a. Disc rotates at constant ω in $-\hat{z}$

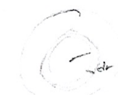
Calculate magnetic dipole of disc

Total charge $Q = \sigma \cdot A$ $\quad A = \pi R^2$

$= \sigma \pi R^2.$

$\mu = IA$

$I = \frac{Q}{T}$



$dA = 2\pi r \, dr$

$dI = \frac{dQ}{dt}$ $\quad$ t → choose 1 period : $\omega \cdot T = 2\pi$

$T = \frac{2\pi}{\omega}$

$dI = \frac{dQ}{2\pi} \omega$

$dQ = \sigma \cdot dA$

$= \sigma 2\pi r \, dr$

$= (2\sigma \pi r \, dr) \frac{\omega}{2\pi} = \sigma \omega r \, dr$

$d\mu = dI \, A = \sigma \omega r \, dr \, (\pi r^2) = \pi \omega \sigma r^3 \, dr$

$$\mu = \int_0^R d\mu = \int_0^R \pi \omega \sigma r^3 dr = \pi \omega \sigma \int_0^R r^3 dr = \pi \omega \sigma \left. \frac{r^4}{4} \right|_0^R = \pi \omega \sigma \frac{R^4}{4}$$

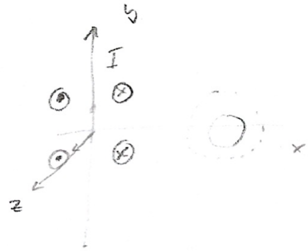
$$= \frac{Q \omega R^2}{4}$$

$$\vec{\mu} = -\frac{Q \omega R^2}{4} \hat{z}$$

b. Line current at y-axis $\rightarrow$ current I

$x_0 >> R$

Calculate force and torque on disc.



$\tau = \vec{\mu} \times \vec{B}$

current density $\Rightarrow$ $\frac{I}{l}$

$\oint B \cdot dl = \mu_0 I \quad \rightarrow \quad B \cdot 2\pi r = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi r} \quad \rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{z})$$

$r = \sqrt{x^2+y^2}$

$$B = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{(x+x_0)^2+y^2}}$$

$x \rightarrow (x+x_0)$

$$\vec{F} = \nabla(\vec{M} \cdot \vec{B}) = \nabla(\mu_z \cdot B_z) = \frac{\mu \mu_0 I}{2\pi}\left[\underbrace{\frac{\partial}{\partial x} B_z}\hat{x} + \frac{\partial}{\partial y} B_z \hat{y}\right]$$

$$\frac{-1}{2}\frac{2(x+x_0)\hat{x}}{[(x+x_0)^2+y^2]^{3/2}} - \frac{1}{2}\frac{2y}{[(x+x_0)^2+y^2]^{3/2}}\hat{y}$$

$$= \frac{-\mu \mu_0 I}{2\pi}\left[\frac{(x+x_0)\hat{x} + y\hat{y}}{[(x+x_0)^2+y^2]^{3/2}}\right]_{\substack{x \to 0,\\ y \to 0}} = \frac{-\mu \mu_0 I}{2\pi}\frac{\hat{x}\, x_0}{(x_0^2)^{3/2}} = \frac{-\mu \mu_0 I}{2\pi}\frac{\hat{x}\, x_0}{(x_0)^3} = \frac{-\mu \mu_0 I}{2\pi x_0^2}\hat{x}$$

$$= \frac{-\pi \sigma R^4}{4} \frac{\mu_0 I}{2\pi x_0^2} = \frac{-\sigma R^4 \mu_0 I}{8 x_0^2}\hat{x}$$

$\tau = \vec{\mu} \times \vec{B} \quad \rightarrow \quad \mu \hat{z} \times B\hat{z} = 0$

$\tau = 0.$