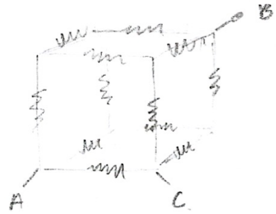
September 2017



a. 12 $1\,\Omega$ resistors in cube. Find effective resistance.

$I_{initial} = 3A$

$V = IR$

$V_i - 1R - \frac{1}{2}R - 1R = V_f$

$V_i - 2\frac{1}{2}R = V_f$

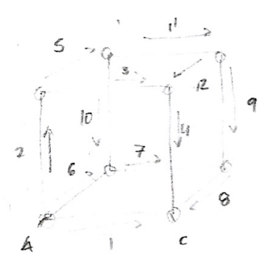
$-2\frac{1}{2}R = V_f - V_i = \Delta V$

$\Delta V = I R_{eff} \Rightarrow -2.5R = 3R_{eff}$

$R_{eff} = \frac{-2.5}{3} R$ $\qquad R = 1\,\Omega \qquad \frac{5}{2}\ \frac{1}{3}$

$R_{eff} = \frac{-5}{6}\,\Omega = \frac{5}{6}\,\Omega$

b. Now it is connected accross C. $\qquad$ 8 nodes



$I_{in} = I_1 + I_2 + I_6$

$I_2 = I_5 + I_3$

$I_5 = I_{11} + I_{10}$

$I_{out} = I_1 + I_4 + I_8$

$I_3 + I_{12} = I_4$

$I_6 + I_{10} = I_7$

$I_{11} = I_{12} + I_9$

$I_7 + I_9 = I_8$

$I_{in} = I_1 + I_2 + I_6 \qquad I_2 = I_{11} + I_{10} + I_4 - I_{12} = I_{12} + I_9 + I_{10} + I_4 - I_{12} = I_9 + I_{10} + I_4$

$= I_9 + I_7 - I_6 + I_4 = I_9 - I_9 + I_8 - I_6 + I_4 = I_8 - I_6 + I_4$

$I_{in} = I_1 + I_8 - I_6 + I_4 + I_6 = I_1 + I_8 + I_4 = I_{out}$ ✓.

$V = I_1 R$

$V = I_2 R + I_3 R + I_4 R$

$V = I_6 R + I_7 R + I_8 R$

$V = (I_2 + I_5 + I_{11} + I_9 + I_8) R$

$I_1 = I_2 + I_3 + I_4 = I_6 + I_7 + I_8 = I_2 + I_5 + I_{11} + I_9 + I_8$

$\Rightarrow I_1 = I_5 + 2I_3 + I_4 = I_5 + 3I_3 + I_{12} = I_2 - I_3 + 3I_3 + I_{12} = I_2 + 2I_3 + I_{12}$

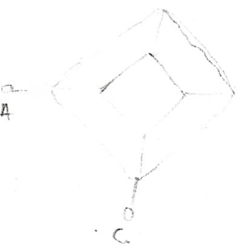
$\Rightarrow I_1 = I_2 + I_2 - I_3 + I_{11} + I_9 + I_8 = 2I_2 - I_3 + I_{12} + 2I_9 + I_8$

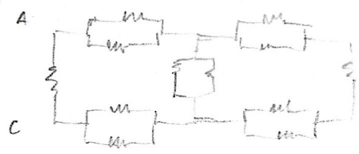
$\Rightarrow I_2 + 2I_3 + I_{12} = 2I_2 - I_3 + I_{12} + 2I_9 + I_8$

$3I_3 = I_2 + 2I_9 + I_8$

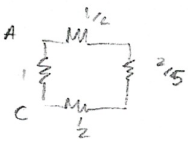
It doesn't ask to solve the whole thing so I am not going to solve that whole thing.

We can't use the same geometry arguments as the current won't split evenly.





$\frac{1}{2}+2=\frac{5}{2}$



$\frac{2}{3}+1=\frac{7}{3}$

$\frac{1}{R_{eff}} = 1+\frac{5}{7}=\frac{12}{7}$

$R_{eff}=\frac{7}{12}$ $\rightarrow$ it decreases