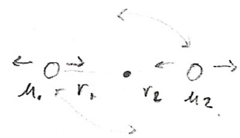
CM-1 September 2019

Diatomic molecule

reduced mass m eq. separation r_0 frequency of radial oscillation ω_0

rotates around an axis perpendicular to vector between molecules.

a. Write expression for energy of system.



$$m = \frac{1}{\frac{1}{\mu_1}+\frac{1}{\mu_2}} = \frac{\mu_1\mu_2}{\mu_1+\mu_2}$$

Center of mass:

$$\frac{\mu_1 r_1 + \mu_2 r_2}{\mu_1+\mu_2} = CoM = \vec{R}$$

Along axis between them.

$$\frac{1}{2}\mu_1 v_1^2 + \frac{1}{2}\mu_2 v_2^2 \rightarrow \frac{m}{2}\left[\frac{\mu_1+\mu_2}{\mu_2}v_1^2 + \frac{\mu_1+\mu_2}{\mu_1}v_2^2\right]$$

$$V_{com} = \frac{\mu_1 V_1 + \mu_2 V_2}{\mu_1+\mu_2}$$

Find v_1 & v_2 $\quad V_1 = \left[\frac{\mu_2}{\mu_1+\mu_2}\right]V_{com}$ & $V_2 = \left[\frac{\mu_1}{\mu_1+\mu_2}\right]V_{com}$

$\downarrow \quad \downarrow$

$\dot{r}_1 \quad \dot{r}_2$

$$T = \frac{m}{2}\left[\frac{\mu_1+\mu_2}{\mu_2}\frac{\mu_2^2}{(\mu_1+\mu_2)^2}V_{com}^2 + \frac{\mu_1+\mu_2}{\mu_1}\frac{\mu_1^2}{(\mu_1+\mu_2)^2}V_{com}^2\right]$$

$V_{com} = \dot{R}$

$$= \frac{m}{2}\dot{R}^2\left[\frac{\mu_2}{\mu_1+\mu_2}+\frac{\mu_1}{\mu_1+\mu_2}\right] = \frac{m}{2}\dot{R}^2$$

Potential $= \frac{1}{2}k(r-r_0)^2$

$= \frac{1}{2}m\omega_0^2(r-r_0)^2$.

$\omega_0 = \sqrt{\frac{k}{m}} \quad \omega_0^2 = \frac{k}{m} \quad k = m\omega_0^2$.

Rotationally:

$T = \frac{1}{2}I\omega^2 \qquad I = \sum \mu_i r_i^2 \rightarrow \mu_1 r_1^2 + \mu_2 r_2^2$

$\mu_1 r_1 = \mu_2 r_2$.

$r_1 = \frac{\mu_2}{\mu_1}r_2$.

$$= \mu_1\left(\frac{\mu_2}{\mu_1}r_2\right)^2 + \mu_2 r_2^2 = \left(\frac{m(\mu_1+\mu_2)^2}{\mu_1^3} + \frac{m(\mu_1+\mu_2)\mu_1^2}{\mu_1^3}\right)r_2^2$$

$= \frac{1}{2}m R^2\dot{\theta}^2$

$$= \frac{m(\mu_1+\mu_2)[\mu_1+\mu_2+\mu_1^2]}{\mu_1^3}r_2^2 = \frac{m(\mu_1+\mu_2)[\mu_1+\mu_2+\mu_1^2]}{\mu_1^3}\frac{\mu_1^2}{(\mu_1+\mu_2)^2}r$$

$\mu_2 = \frac{m(\mu_1+\mu_2)}{\mu_1}$

$r_2 = \frac{\mu_1}{\mu_1+\mu_2}r$.

$$= \frac{m(\mu_1+\mu_2+\mu_1^2)}{\mu_1(\mu_1+\mu_2)}r^2$$

ignore this math

Right process:

$$\rightarrow \mu_1\left(\frac{\mu_2}{\mu_1}\frac{\mu_1}{\mu_1+\mu_2}r\right)^2 + \mu_2\left(\frac{\mu_1}{\mu_1+\mu_2}r\right)^2 = \left[\mu_1\frac{\mu_2^2}{(\mu_1+\mu_2)^2} + \frac{\mu_1^2\mu_2}{(\mu_1+\mu_2)^2}\right]r^2$$

$$= \left(\frac{\mu_1\mu_2^2+\mu_1^2\mu_2}{(\mu_1+\mu_2)^2}\right)r^2 = \left(\frac{\mu_1\mu_2(\mu_2+\mu_1)}{(\mu_2+\mu_1)^2}\right)r^2 = mr^2$$

$E_{TOTAL} = \frac{m}{2}\dot{r}^2 + \frac{1}{2}m\omega^2(r-r_0)^2 + \frac{1}{2}mr^2\dot{\theta}^2$

. Find first order correction to equilibrium for small L.

$r \to r_0 + \delta r$ for $\delta r \ll r_0$

angular momentum: $\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = P_\theta$

$\mathcal{L} = \frac{m}{2}\dot{r}^2 - \frac{1}{2}m\omega_0^2(r-r_0)^2 + \frac{1}{2}mr\dot{\theta}^2$

$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mr^2\dot{\theta}$

$L = m(r_0+\delta r)^2\dot{\theta} = mr_0^2\left(1+\frac{\delta r}{r_0}\right)^2\dot{\theta} \approx mr_0^2\left(1+2\frac{\delta r}{r_0}\right)\dot{\theta}$

$\frac{L}{mr_0^2\dot{\theta}} = 1 + \frac{2\delta r}{r_0} \rightarrow \frac{L}{mr_0^2\dot{\theta}} - 1 = \frac{2\delta r}{r_0}$

$\delta r = \frac{L}{2mr_0\dot{\theta}} - \frac{r_0}{2}$

. Find first order correction to frequency of radial oscillation

$\omega \to \omega_0 + \delta\omega$ $r \to r_0 + \delta r$

$\frac{1}{2}m\omega_0^2(r-r_0)^2 \rightarrow \frac{1}{2}m(\omega_0+\delta\omega)^2(r-r_0)^2 \approx \frac{1}{2}m\omega_0^2\left(1+2\frac{\delta\omega}{r_0}\right)\underbrace{(r_0+\delta r-r_0)^2}_{(\delta r)^2}$

$\Rightarrow \frac{1}{2}m\omega_0\left(1+\frac{2\delta\omega}{r_0}\right)\left(\frac{L}{2mr_0\dot{\theta}} - \frac{r_0}{2}\right)^2$

→ Not sure about this