SM-2 January 2013

Speed distribution

$f(v) \propto v^2 e^{-mv^2/2k_BT}$

# of molecules with v that hit wall is proportional to $\underline{v}$ and $\underline{f(v)}$ $\rightarrow N \propto v f(v)$

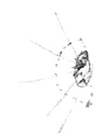
a. Write speed distribution of particles that escape $F(v)$.

Hit wall $N \rightarrow v f(v)$ Don't hit wall $(1-N) \rightarrow 1 - v f(v)$

$\frac{PV}{N} = k_B T$ $f(v) \propto v^2 e^{-mv^2 N/2PV}$

flux: $\Phi \rightarrow$ depends on N, surface S and time T.

$\Phi = \frac{dN}{dS\,dt}$ N depends on the angle particles are coming from

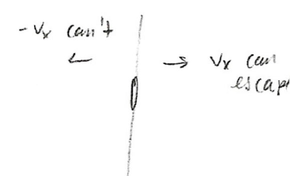


$dN = n\,dV$ for $n = \frac{N}{V}$ $\rightarrow dN = n\,dS\,v\cos\theta\,dt$

Particles leaving:

$$\underbrace{dN f(v)\,dv}_{n\,dS\,v\cos\theta\,dt} \cdot \underbrace{\frac{d\Omega}{4\pi}}_{\frac{\sin\theta\,d\theta\,d\phi}{4\pi}}$$

$dV = dS\,v\cos\theta\,dt$



Particles will have same speed distribution but only for $v_x > 0$

$$F(v_z) = \begin{cases} f(v) & \text{for } v > 0 \\ 0 & \text{for } v \leq 0 \end{cases}$$

while $f(v_x)$ and $f(v_y)$ remain the same.

b. Calculate average energy of molecules that escape.

$$\langle v_z \rangle = \frac{\int_0^\infty dv\, v^2\, F(x)}{\int_0^\infty dv\, F(x)} = \frac{\int_0^\infty dv\, v^4 e^{-mv^2/2PV}}{\int_0^\infty dv\, v^2 e^{-mv^2/2PV}} = \frac{\frac{3\sqrt{\pi}}{8\,(m/2PV)^{5/2}}}{\frac{\sqrt{\pi}}{4\,(m/2PV)^{3/2}}}$$

$$= \frac{3\,(m/2PV)^{3/2}}{2\,(m/2PV)^{5/2}} = \frac{3}{2} \cdot \frac{2PV}{m} = 3\,\frac{PV}{m}$$

$PV = k_B T$

$$\frac{1}{2}\, m \left(v_z^2 + v_x^2 + v_y^2\right) = \frac{1}{2}\left(3PV + 2k_BT\right) = \frac{5}{2}\, k_B T$$

→ apparently should be $2\, k_B T$ so not sure about this.