QM-2 January 2014

Consider 3 particles with spin $S_1 = S_2 = S_3 = 1$

a. Find dimension of hilbert space:

$$1 \otimes 1 \otimes 1 = (2S_1+1)(2S_2+1)(2S_3+1) = 3^3 = 27$$

b. What are the possible values of angular momentum?

0, 1, 2, 3

c. How many linearly independent states for each value of total angular momentum?

$$1 \otimes (1 \otimes 1) = 1 \otimes (0 \oplus 1 \oplus 2) = (1 \otimes 0) \oplus (1 \otimes 1) \oplus (1 \otimes 2)$$

$$= 1 \oplus (0 \oplus 1 \oplus 2) \oplus (1 \oplus 2 \oplus 3)$$

$$= \underset{1}{\underset{\downarrow}{0}} \oplus \underset{3}{\underset{\downarrow}{1}} \oplus \underset{3}{\underset{\downarrow}{1}} \oplus \underset{3}{\underset{\downarrow}{1}} \oplus \underset{5}{\underset{\downarrow}{2}} \oplus \underset{5}{\underset{\downarrow}{2}} \oplus \underset{7}{\underset{\downarrow}{3}}$$

1 state momentum 0, 9 states momentum 1, 10 states momentum 2 and 7 states momentum 3.

d. Find the eigenvalue of:

$$\hat{H} = J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1)$$

$$\vec{S}^2 = (\vec{S}_1 + \vec{S}_2 + \vec{S}_3) \cdot (\vec{S}_1 + \vec{S}_2 + \vec{S}_3) = S_1^2 + S_1 \cdot S_2 + S_1 \cdot S_3 + S_2^2 + S_2 \cdot S_1 + S_2 \cdot S_3 + S_3^2 + S_3 \cdot S_1 + S_3 \cdot S_2$$

$$= S_1^2 + S_2^2 + S_3^2 + 2(S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1)$$

$$\hat{H} = \Big[\underset{S(S+1)}{\underset{\downarrow}{\vec{S}^2}} - (S_1^2 + S_2^2 + S_3^2)\Big]\frac{J}{2}$$

$$S_1^2 = S_2^2 = S_3^2 = S_1(S_1+1) = 2$$

$$S(S+1) = \begin{cases} 0 & \text{for } S=0 \rightarrow \text{non-degenerate} \\ 2 & \text{for } S=1 \rightarrow \text{9-fold degeneracy} \\ 6 & \text{for } S=2 \rightarrow \text{10-fold degeneracy} \\ 12 & \text{for } S=3 \rightarrow \text{7-fold degeneracy} \end{cases}$$

$$= \frac{J}{2}\left[S(S+1) - 6\right]$$

$$H = J \begin{cases} -3 & \text{1 time} \\ -2 & \text{9 times} \\ 0 & \text{10 times} \\ 3 & \text{7 times} \end{cases}$$