SM-2 January 2015

a. Calculate equation of state
non-interacting 2D Bose gas. mass m.

$$Z_1 = \sum_N e^{-\beta(E_1-\mu)N} = \frac{1}{1-e^{-\beta(E_1-\mu)}} \qquad E_1 = \frac{p^2}{2m}$$

$$Z_T = \prod_n \frac{1}{1-e^{-\beta(E_n-\mu)}} = \prod_n \frac{1}{1-z e^{-\beta E_n}} \qquad z = e^{\beta\mu}$$

$$= \prod_n \frac{1}{1-z\, e^{-\beta p_n^2/2m}}$$

b. Calculate average number of particles over unit area.

$$N = \frac{1}{\beta}\frac{\partial}{\partial\mu}\ln Z = \frac{1}{\beta}\frac{\partial}{\partial\mu}\sum_n \ln\left(\frac{1}{1-z e^{-\beta p_n^2/2m}}\right) = \sum_n \frac{e^{-\beta E_n}}{1-z e^{-\beta E_n}} = \sum_n \frac{1}{z^{-1}e^{\beta E_n}-1}$$

Take $\sum_n \sim \int d^2 n = 2\pi \int n\, dn$ $\qquad KL = 2\pi n \qquad n = \frac{KL}{2\pi}$ $\qquad A = L^2$

$$\to \frac{A}{2\pi}\int dK\, K \to \frac{A}{2\pi\hbar^2}\int dp\, p \qquad \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m} \qquad k = \frac{p}{\hbar}$$

$$N = \frac{A}{2\pi\hbar^2}\int \frac{p\, dp}{z^{-1} e^{\beta p^2/2m}-1}$$

$$\frac{N}{A} = \frac{1}{2\pi\hbar^2}\int_{-\infty}^{\infty} \frac{p}{z^{-1} e^{\beta p^2/2m}-1}\, dp \qquad E = \frac{p^2}{2m} \qquad dE = \frac{p}{m}dp$$

c. Show no condensation happens.

Take $\mu \to 0$.

$p\,dp \sim dE$

$$\frac{N}{A} = C \int_0^\infty \frac{dE}{e^{\beta E} - 1}$$

$$\approx C \int_0^\infty \frac{dE}{1 + \beta E - 1} = \frac{C}{\beta} \int_0^\infty \frac{1}{E}\, dE = \frac{C}{\beta} \int_{-\infty}^{\infty} \frac{p\,dp}{p^2} = \frac{C}{k_B} \underbrace{\int_{-\infty}^{\infty} \frac{T}{p}\, dp}_{\text{diverges for } T \neq 0}$$

For $d = 2$, $T_c \Rightarrow 0$. No condensation.