

QM-1

January 2016

Spin 0 particle in 3D

$$V(\vec{r}) = \frac{k}{2} r^2$$

a. Find eigenvalues and determine degeneracy of first five.

$$H = \frac{\vec{p}^2}{2m} + \frac{k}{2} r^2 = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{k}{2} (x^2 + y^2 + z^2)$$

$$H_x = \frac{p_x^2}{2m} + \frac{k}{2} x^2 \rightarrow \text{harmonic oscillator}$$

$$H = \frac{p^2}{2m} + 2m\omega^2 x^2$$

$$E_x = (n + \frac{1}{2}) \hbar \omega$$

$$= (n + \frac{1}{2}) \sqrt{\frac{k}{m}} \hbar$$

$$\frac{1}{2} m \omega^2 = \frac{k}{2}$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$H = H_x + H_y + H_z \Rightarrow E = \sqrt{\frac{k}{m}} \hbar \left[n_x + n_y + n_z + \frac{3}{2} \right]$$

$$E_0 = \frac{3}{2} \sqrt{\frac{k}{m}} \hbar$$

- no degeneracy

$$E_1 = \frac{5}{2} \sqrt{\frac{k}{m}} \hbar$$

- 3 fold degenerate

$$E_2 = \frac{7}{2} \sqrt{\frac{k}{m}} \hbar$$

- 6 fold degenerate

$$E_3 = \frac{9}{2} \sqrt{\frac{k}{m}} \hbar$$

- 10 fold degenerate

$$E_4 = \frac{11}{2} \sqrt{\frac{k}{m}} \hbar$$

- 15 fold degenerate

 $2x, 2y, 2z, x+y, x+z, y+z$

$$\left. \begin{array}{l} 4 - 3 \\ 31 - 6 \\ 22 - 3 \\ 211 - 3 \end{array} \right\} 15$$

b. Suppose there are 6 non-interacting particles. Find ground state

i) $\frac{1}{2}$

$$\# = (1+25) = 2$$

 E_0 E_1 E_2 $\uparrow \downarrow$ $\uparrow \downarrow$ $\uparrow \downarrow$

$$\rightarrow \left(\frac{3}{2} \times 2 + \frac{5}{2} \times 2 + \frac{7}{2} \times 2 \right) \sqrt{\frac{k}{m}} \hbar = (3+5+7) \sqrt{\frac{k}{m}} \hbar = 15 \sqrt{\frac{k}{m}} \hbar$$

ii) 1

boson!

 E_0 $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

$$\rightarrow \left(\frac{3}{2} \times 6 \right) \sqrt{\frac{k}{m}} \hbar = 9 \sqrt{\frac{k}{m}} \hbar$$

iii) $\frac{3}{2}$

$$\# = (1+25) = 1+3 = 4$$

 E_0 E_1 $\uparrow \downarrow \uparrow \downarrow$ $\uparrow \downarrow$

$$\rightarrow \left(\frac{3}{2} \times 4 + \frac{5}{2} \times 2 \right) \sqrt{\frac{k}{m}} \hbar = (6+5) \sqrt{\frac{k}{m}} \hbar$$