QM-2 January 2018

Check this out

Q.M. system $|1\rangle$ & $|2\rangle$

$E_1 = E_2 \rightarrow$ degenerate

Perturbation $V = \begin{bmatrix} 0 & V_{12} \\ V_{21} & 0 \end{bmatrix}$

$V_{21} = V_{12}^*$

at $t=0 \rightarrow$ system in state $|1\rangle$

Find amplitude for finding system at each state at later time

$|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $|2\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\det\begin{pmatrix} -\lambda & V_{12} \\ V_{21} & -\lambda \end{pmatrix} = 0 \Rightarrow (\lambda^2 - |V_{12}|^2) = 0 \Rightarrow \lambda^2 = |V_{12}|^2 \qquad \lambda = \pm\sqrt{V_{12}V_{21}}$$

$\lambda_1 = \sqrt{V_{12}V_{21}}$

$$\begin{bmatrix} \sqrt{V_{12}V_{21}} & V_{12} \\ V_{21} & \sqrt{V_{12}V_{21}} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \sqrt{V_{12}V_{21}}\, x_1 + V_{12} x_2 = 0$$

$$\sqrt{\frac{V_{21}}{V_{12}}}\, x_1 = x_2 \Rightarrow \vec{V_1} = \begin{bmatrix} 1 \\ \sqrt{\frac{V_{12}}{V_{21}}} \end{bmatrix}$$

$\lambda_2 = -\sqrt{V_{12}V_{21}}$

$$\vec{V_2} = \begin{bmatrix} 1 \\ -\sqrt{\frac{V_{12}}{V_{21}}} \end{bmatrix}$$

$$|1\rangle \rightarrow |1\rangle + \begin{bmatrix} \sqrt{V_{12}V_{21}} \\ V_{12} \end{bmatrix}$$

$$|2\rangle = |1\rangle - \begin{bmatrix} \sqrt{V_{12}V_{21}} \\ -V_{12} \end{bmatrix}$$