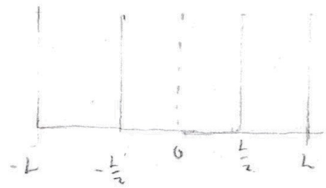
QM-4 January 2018

1D particle in infinite well

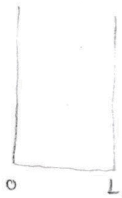
Instant expansion

mass m.



a. Find eigenfunctions & eigenstates in initial configuration

Move it from 0 to L



$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi \quad \rightarrow \quad \frac{d^2}{dx^2}\psi = \frac{-2m}{\hbar^2}E\psi$$

$$\psi = A\cos(kx) + B\sin(kx)$$

$$\psi = B\sin(kx)$$

$$\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)$$

$k = \frac{\sqrt{2mE}}{\hbar}$

$A \to 0.$

$\langle\psi|\psi\rangle = B^2\int_0^L \psi^*\psi\,dx = 1$

$\frac{1}{B^2} = \int_0^L \sin^2(kx)dx$

$\frac{1}{B^2} = \frac{2}{K} \qquad B = \sqrt{\frac{2}{L}}$

$\sin(Lk) = 0$

$Lk = n\pi$

$k = \frac{n\pi}{L}$

$\frac{n\pi}{L} = \frac{\sqrt{2mE}}{\hbar}$

$\frac{n^2\pi^2\hbar^2}{L^2} = 2mE$

Eigenvalues. $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$

Shift it back

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left[\frac{n\pi}{L}\left(x+\frac{L}{2}\right)\right]$$

energy remains the same:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Ground state:

$$\psi_0 = \sqrt{\frac{2}{L}}\sin\left[\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right]$$

$$E_0 = \frac{\pi^2\hbar^2}{2mL^2}$$

First excited state:

$$\psi_1 = \sqrt{\frac{2}{L}}\sin\left[\frac{2\pi}{L}\left(x+\frac{L}{2}\right)\right]$$

$$E_1 = \frac{4\pi^2\hbar^2}{2mL^2}$$

$\sin(kx+\pi)$ is a $-\sin$ I think

$$\psi_1 = -\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi}{L}x\right)$$

b. Find eigenvalues and functions after expansion

$L \to 2L$

$B' = \sqrt{\frac{1}{L}}$

$\psi' = \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi}{2L} x\right)$

$K = \frac{\sqrt{2mE}}{\hbar}$

$2KL = n\pi$

$K = \frac{n\pi}{2L}$

$\frac{\hbar^2 n^2 \pi^2}{4L^2} = 2mE$

$E' = \frac{\hbar^2 n^2 \pi^2}{8mL^2}$

shift back:

$\psi' = \sqrt{\frac{1}{L}} \sin\left[\frac{n\pi}{2L}(x+L)\right]$

Ground state.

$\psi_0' = \sqrt{\frac{1}{L}} \sin\left(\frac{\pi}{2L}(x+L)\right)$

$E_0' = \frac{\hbar^2 \pi^2}{8mL^2}$

Excited state

$\psi_1' = \sqrt{\frac{1}{L}} \sin\left[\frac{\pi}{L}(x+L)\right]$

$E_1' = \frac{\hbar^2 \pi^2}{2mL^2}$

c. Find probability it ends in E_1' if originally in E_0

$P = |\langle \psi_1' | \psi_0 \rangle|^2$

$P = \int_{-\infty}^{\infty} \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x + \frac{\pi}{2}\right) \sqrt{\frac{1}{L}} \sin\left(\frac{\pi}{L}x + \pi\right) dx$

$= \frac{\sqrt{2}}{L} \int_{-\infty}^{\infty} -\cos\left(\frac{\pi}{L}x\right) \sin\left(\frac{\pi}{L}x\right) dx = -\frac{\sqrt{2}}{L} \frac{1}{2} \sin^2\left(\frac{\pi}{L}x\right) \Big|_{-L}^{L} = 0.$