QM-4 January 2019

Particle charge e

mass m

$$\vec{E} = \begin{bmatrix} 0 \\ 0 \\ E \end{bmatrix} , \quad \vec{B} = \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix}$$

a. Write Schrodinger equation using gauge $\vec{A} = \begin{bmatrix} Bz \\ 0 \\ 0 \end{bmatrix}$, $\phi = -Ez$

$$\hat{H} = \frac{(\vec{p} - \frac{e}{c}\vec{A})^2}{2m} - eEz \Rightarrow \frac{(\vec{p} - \frac{e}{c}\vec{A})^2}{2m}\Psi - eEz\Psi = \mathcal{E}\Psi$$

$\mathcal{E} \to$ energy

$$\frac{p_z^2 + p_y^2 + (p_x - \frac{Be}{c}z)^2}{2m}\Psi - eEz\Psi = \mathcal{E}\Psi$$

b separate variables & reduce to 1D problem

$\Psi(x, y, z) = \Psi_x \Psi_y \Psi_z$

$\mathcal{E} = \mathcal{E}_x + \mathcal{E}_y + \mathcal{E}_z$

$H_y = \frac{p_y^2}{2m} \Rightarrow \frac{p_y^2}{2m}\Psi = \mathcal{E}_y\Psi$ $\quad \Psi_y = e^{ik_y y}$ $\quad k_y = \frac{\sqrt{2\mathcal{E}_y m}}{\hbar}$

Similarly for p_x.

$\Psi_x = e^{ik_x x}$ $\quad k_x = \frac{\sqrt{2\mathcal{E}_x m}}{\hbar}$

$p_y^2 = 2m\mathcal{E}_y = \hbar^2 k_y^2$

$p_x = \hbar k_x$

$$\Rightarrow \frac{p_z^2 + \hbar^2 k_y^2 + (\hbar k_x - \frac{Bez}{c})^2}{2m}\Psi_z - eEz\Psi_z = \mathcal{E}\Psi_z$$

$$\left[\frac{p_z^2}{2m} + \frac{(\hbar k_x - \frac{Be}{c}z)^2}{2m} - eEz\right]\Psi_z = (\mathcal{E} - \hbar^2 k_y^2)\Psi_z$$

$$\left[\frac{p_z^2}{2m} + \frac{B^2e^2}{c^2 2m}z^2 - \left(\frac{\hbar k_x Be}{mc} + eE\right)z\right]\Psi_z = (\mathcal{E} - \hbar^2 k_y^2 - \hbar^2 k_x^2)\Psi_z$$

$$\frac{p_z^2}{2m} + \frac{B^2e^2}{2mc^2}\left[z^2 - \frac{2mc^2}{B^2e^2}\left(\frac{\hbar k_x Be}{mc} + Ee\right)z\right] =$$

$$z^2 - 2\left(\frac{\hbar k_x c}{Be} + \frac{mc^2 E}{B^2 e}\right)z$$

$z^2 - \frac{\sqrt{a}}{2}z + a$

$(z-a)^2 = z^2 - 2a + a^2$

$$\frac{p_z^2}{2m} + \frac{B^2e^2}{2mc^2}(z - z_0)^2 - \left(\frac{B\hbar k_x c + mc^2 E}{B^2 e}\right)^2 = (\mathcal{E} - \hbar^2 k_y^2 - \hbar^2 k_x^2)$$

$z_0 = \frac{B\hbar k_x c + mc^2 E}{B^2 e}$

$$\frac{p_z^2}{2m} + \underbrace{\frac{B^2e^2}{2mc^2}}_{\frac{1}{2}m\omega_c^2}\underbrace{(z - z_0)^2}_{\tilde{z}} = \underbrace{\mathcal{E} - \hbar^2 k_y^2 - \hbar^2 k_x^2 + \left(\frac{B\hbar k_x c + mc^2 E}{Be^2}\right)}_{\tilde{\mathcal{E}}}$$

$$\omega_c^2 = \frac{B^2e^2}{m^2c^2}$$

c. Find $\langle v \rangle$ in x direction of any eigenstate.

$$\left\langle \frac{d}{dt} x \right\rangle = \frac{-i}{\hbar} \langle [x, H] \rangle$$ ehernfest theorem.

$\downarrow$

look only at H_x since x commutes with p_y & p_z

$$= \frac{-i}{\hbar} \left\langle \left[x, \frac{p_x{}^2}{2m} - \frac{eB}{mc} z p_x \right] \right\rangle$$

$$= \frac{-i}{\hbar} \left(\frac{1}{2m} \underbrace{\langle [x, p_x{}^2] \rangle}_{2i\hbar p_x} - \frac{eB}{mc} z \underbrace{\langle [x, p_x] \rangle}_{i\hbar} \right)$$

$$[x, p_x{}^2] = p_x [x, p_x] + [x, p_x] p_x$$

$$= \frac{p_x}{m} - \frac{eBz}{mc}$$

$$\langle \dot{x} \rangle = \left\langle \frac{p_x}{m} - \frac{eBz}{mc} \right\rangle = \frac{\langle p_x \rangle}{m} - \frac{eB}{mc} \langle z \rangle$$