SM-1 September 2013

Quantum particle spin S in magnetic field $\vec{B}$

$$H = -\vec{\mu}\cdot\vec{B} \quad \text{where } \vec{\mu} = \mu\vec{S}$$

Find magnetic susceptibility at zero field and temperature T

$$H = -\mu\vec{S}\cdot\vec{B}$$

$$Z_1 = e^{-\beta H} = \sum_{s=-S}^{S} e^{-\beta\mu s B} = \sum_{s=0}^{S}\left(e^{-\beta\mu s B} + e^{\beta\mu s B}\right) = \underbrace{e^{-\beta\mu B S} + e^{\beta\mu B S}}_{2\cosh(\beta\mu B s)} + \underbrace{e^{-\beta\mu B(S-1)} + e^{\beta\mu B(S-1)}}_{2\cos(\beta\mu s(s-1))} + \dots$$

↳ this sum equals:

$$\frac{\sinh[(2s+1)\mu B\beta/2]}{\sinh[\mu B\beta/2]} \quad +1 \to \text{integer spi}$$

$$\to \sum_{s=0}^{S}\cosh(\beta\mu s B)$$

$$F = -\frac{1}{\beta}\ln Z \quad \to \text{Free energy}$$

$$= -\frac{1}{\beta}\ln\sum_{s=-S}^{S} e^{-\beta\mu s B}$$

$$m = -\frac{\partial F}{\partial B} = \frac{1}{\beta Z}\frac{\partial}{\partial B}Z = \frac{1}{\beta Z}\sum_{-S}^{S}(-\beta\mu s)e^{-\beta\mu s B}$$

$$= -\frac{\mu}{Z}\sum_{-S}^{S} s\, e^{-\beta\mu B s}$$

$$\chi = \frac{\partial}{\partial B}m\Big|_{B\to 0}$$

$$= -\mu\left[\left(\frac{\partial}{\partial B}\frac{1}{Z}\right)\sum_{-S}^{S} s e^{-\beta\mu B s} + \frac{1}{Z}\frac{\partial}{\partial B}\sum_{-S}^{S} s e^{-\beta\mu B s}\right]$$

$$Z\Big|_{B\to 0} = 2s+1$$

$$= -\mu\left[\frac{-1}{Z^2}\sum_{-S}^{S}(-\beta\mu s)e^{-\beta\mu s B} + \frac{1}{Z}\sum_{-S}^{S}(-\beta\mu s^2)e^{-\beta\mu B s}\right]$$

$$= -\mu\left[\frac{\mu B}{Z^2}\underbrace{\sum_{-S}^{S} s}_{=0} + \frac{-\mu\beta}{Z}\sum_{-S}^{S} s^2\right]_{B\to 0}$$

$$= \frac{\beta\mu}{2s+1}\sum_{-S}^{S} s^2 = \frac{\beta\mu}{2s+1}\cdot\frac{1}{3}s(s+1)(2s+1)$$

$$= \frac{\beta\mu\, s(s+1)}{3}$$