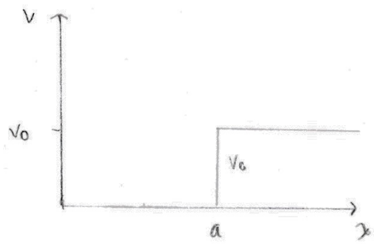
QM-4 September 2018

Particle of mass m

$$V(x) = \begin{cases} \infty & \text{for } x<0 \\ 0 & \text{for } 0<x<a \\ V_0 & \text{for } x>a \end{cases} \quad \rightarrow V_0 > 0$$



a. $E = \frac{\hbar^2 k^2}{2m}$ → bound state $k = \frac{\sqrt{2mE}}{\hbar}$

$E = V_0 - \frac{\hbar^2 K^2}{2m}$ $K = -\frac{\sqrt{2m(E-V_0)}}{\hbar}$

Give equation that determines possible values of E.

Region I Region II

$A\sin(kx)$ Be^{-Kx}

At $x=a$:

$$\psi_I = \psi_{II} \quad \text{and} \quad \frac{d}{dx}\psi_I = \frac{d}{dx}\psi_{II}$$

$$\Rightarrow A\sin(ka) = Be^{-Ka}$$

$$Ak\cos(ka) = -BKe^{-Ka} \Rightarrow Ak\cos(ka) = -KA\sin(ka)$$

$$\tan(ka) = \frac{-k}{K} \rightarrow \frac{1}{K} = -\frac{\tan(ka)}{k} \quad \text{or} \quad K = -\cot(ka)\,k$$

$$E(k) = \frac{\hbar^2 k^2}{2m} \rightarrow \text{region I}$$

$$E(k) = V_0 + \frac{\hbar^2 k^2}{2m}\cot^2(ka) \rightarrow \text{region II}$$

$$\tan\left(\frac{\sqrt{2mE}}{\hbar}a\right) = \sqrt{\frac{2mE}{2m(E-V_0)}} = \sqrt{\frac{E}{E-V_0}}$$

b. Condition for bound state.

$E < V_0$

$$\tan\left(\frac{\sqrt{2mE}}{\hbar}a\right) = \sqrt{\frac{E}{E-V_0}}$$

$$\tan^2\left(\frac{\sqrt{2mEa^2}}{\hbar}\right)(E-V_0) = E < V_0$$

$$\tan^2\left(\frac{\sqrt{2mEa^2}}{\hbar}\right)E < V_0\left(1+\tan^2\left(\frac{\sqrt{2mEa^2}}{\hbar}\right)\right)$$

$$\frac{\tan^2\left(\frac{\sqrt{2mEa^2}}{\hbar}\right)}{\underbrace{1+\tan^2\left(\frac{\sqrt{2mEa^2}}{\hbar}\right)}_{\sec^2}} < \frac{V_0}{E}$$

$$\frac{\sin^2}{\cos^2}\cdot\frac{\cos^2}{1} = \sin^2$$

$$\sin^2\left(\frac{\sqrt{2mEa^2}}{\hbar}\right) < \frac{V_0}{E}$$

$$\sin^2(ka) < \frac{V_0}{E}$$

$$\sin^2(ka) > \frac{E}{V_0}$$

$[0, 1]$

$a \leq \pi \quad \frac{\pi}{2}$

$$E < V_0 \sin^2(ka)$$

c. Energy levels at $V_0 = \infty$

→ infinite square well

Region I.

$\Psi = A\sin(kx) \rightarrow A\sin(ka) = 0 \Rightarrow ka = n\pi \rightarrow \frac{\sqrt{2mE}}{\hbar} = \frac{n\pi}{a}$

$$E = \frac{1}{2m}\left(\frac{\hbar n\pi}{a}\right)^2$$